ALGORITHMS TO FIT STEM CROSS-SECTIONS UNDER PARTIAL OCCLUSION

JOHN KERSHAW
TING-RU YANG
UNIVERSITY OF NEW BRUNSWICK

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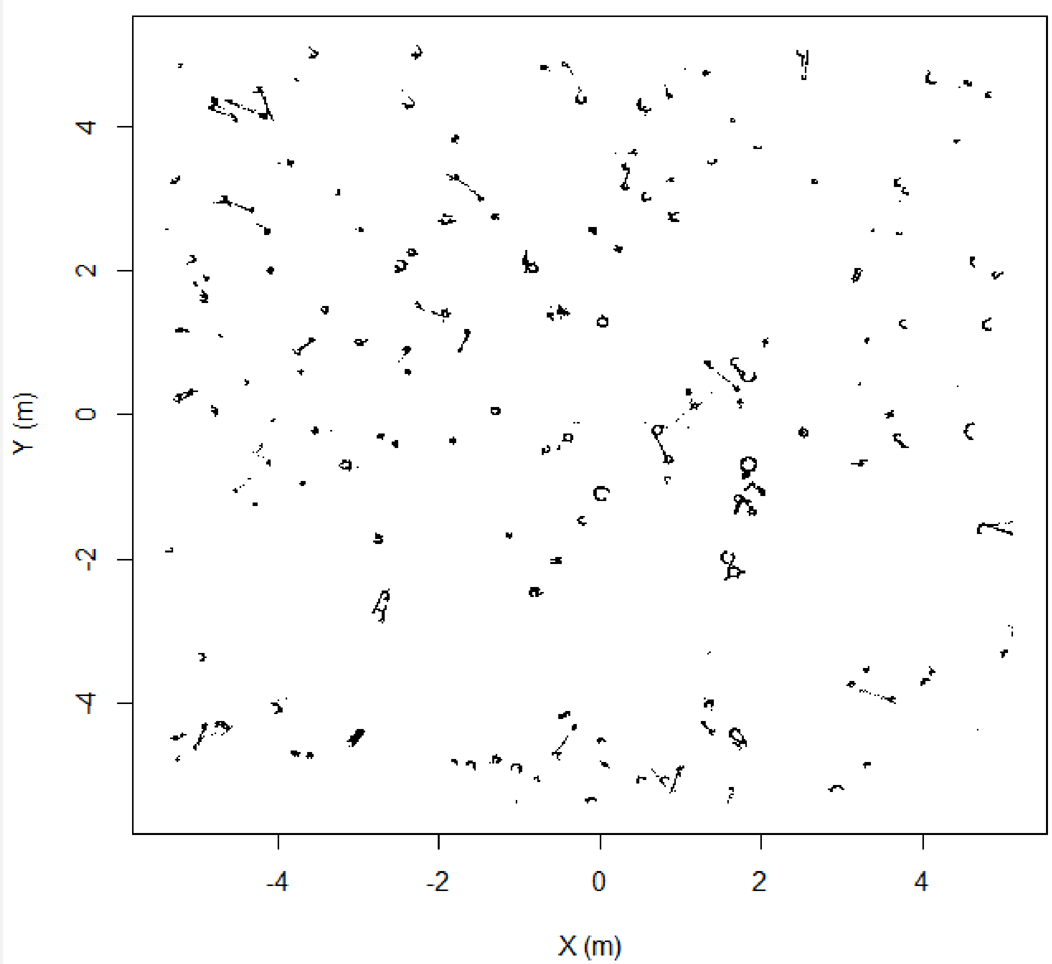
NEWFOUNDLAND SPACING TRIALS

- Established early 1960s
- Control, 1.2, 1.8, 2.4, and 3.0 m spacings
- Balsam Fir
- 3 sites
- 3 replicates per site

TERRESTRIAL LIDAR SCANS



ELLIPSE FITTING ALGORITHM

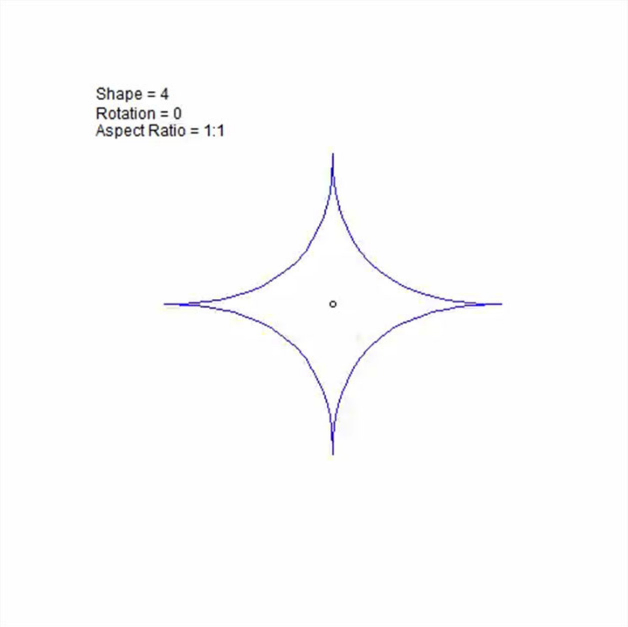
- SOMENS/NEMO 2018, Blacksburg
- Generally fit fairly well
- Produced some extremely bad fits in some cases
  - Over/under estimation depending on curvature, missingness, and aspect ratio

- Funding shifted
- Got sidetracked with other projects and set it aside
- Then…….. COVID19 gave me way too much home time…….

THE SUPERELLIPSE

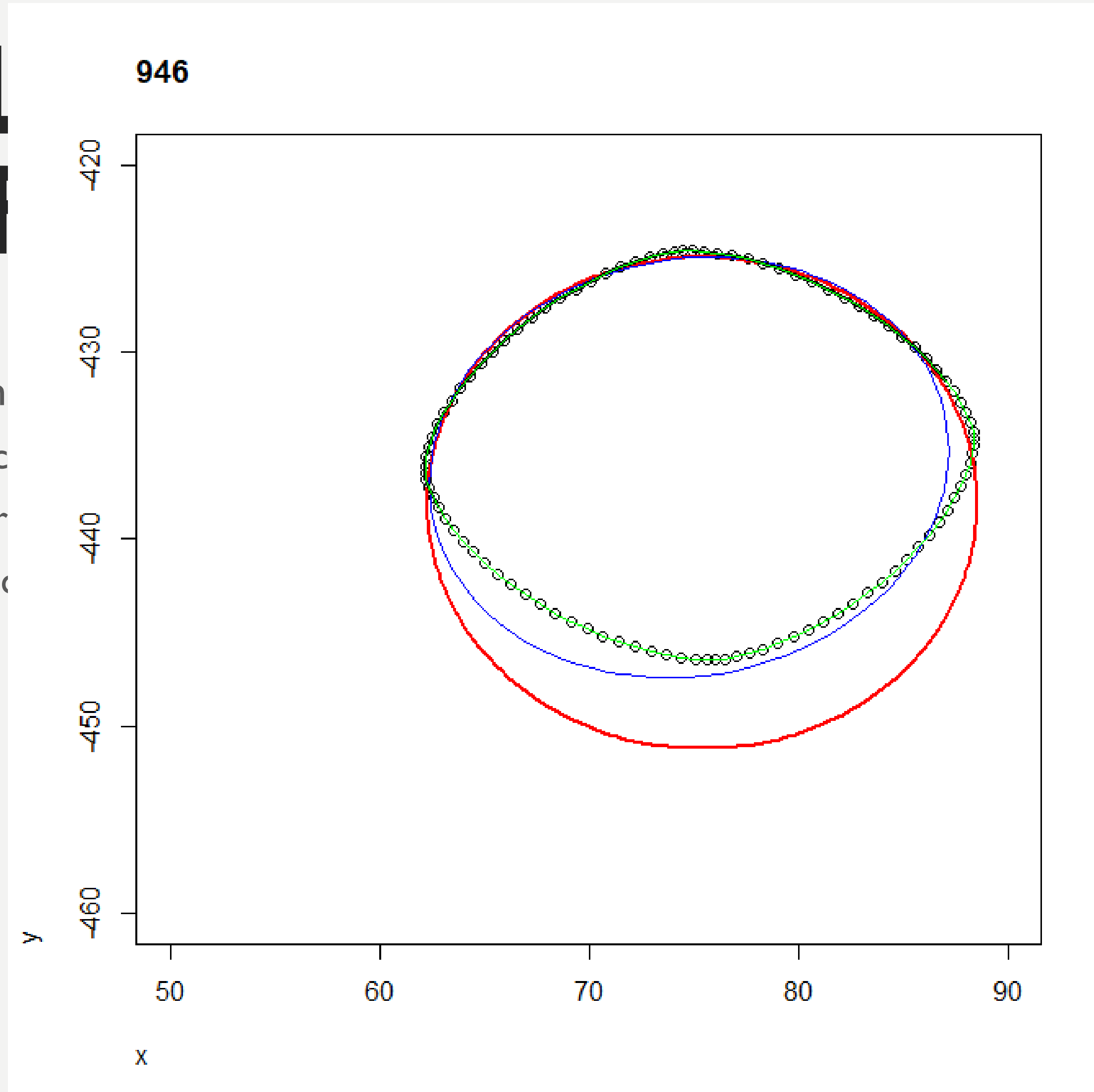
$$Q_0(x,y)=\left[\frac{(x-x_O)\cos\theta-(y-y_O)\sin\theta}{a}\right]^{2/n}$$

$$+\left[\frac{(y-y_O)\cos\theta+(x-x_O)\sin\theta}{b}\right]^{2/n}-1$$



CIRCL
SUPEF ... IES

- Minimization
  - Algebraic
  - Geometr
  - Perpendic
  - ….



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03

DERIVING A SUPERELLIPSE FITTING ALGORITHM

$$Q_0(x,y)=\left[\frac{(x-x_O)\cos\theta-(y-y_O)\sin\theta}{a}\right]^{2/n}+\left[\frac{(y-y_O)\cos\theta+(x-x_O)\sin\theta}{b}\right]^{2/n}-1$$

LET:

$$u=(x-x_O)\cos\theta-(y-y_O)\sin\theta \qquad v=(y-y_O)\cos\theta+(x-x_O)\sin\theta$$

THEN:

$$Q_0(u,v)=\left[\frac{u}{a}\right]^{2/n}+\left[\frac{v}{b}\right]^{2/n}-1 \quad \Rightarrow \quad \begin{cases} u(\phi)=|\cos\phi|^{2/n}\cdot a\cdot \operatorname{sgn}(\cos\phi) \\ v(\phi)=|\sin\phi|^{2/n}\cdot b\cdot \operatorname{sgn}(\sin\phi) \end{cases}$$

DERIVING A SUPERELLIPSE FITTING ALGORITHM

NOW LET:

$$\left\{ \begin{aligned} X &= \left[\frac{(x - x_O)\cos\theta - (y - y_O)\sin\theta}{a} \right]^{1/n} \\ Y &= \left[\frac{(y - y_O)\cos\theta + (x - x_O)\sin\theta}{b} \right]^{1/n} \end{aligned} \right\}$$

THEN:

$$Q_0'(X, Y) = X^2 + Y^2 - 1 = Q_0(x, y) \quad \text{=>} \quad \left\{ \begin{aligned} X &= \cos\nu \\ Y &= \sin\nu \end{aligned} \right\}$$

DERIVING A SUPERELLIPSE FITTING ALGORITHM

GRADIENT:

$$Q_1\left(x, y\right) = 2XX_t' + 2YY_t'$$

CURVATURE:

$$Q_2\left(x, y\right) = XX_t'' + YY_t'' + 1$$

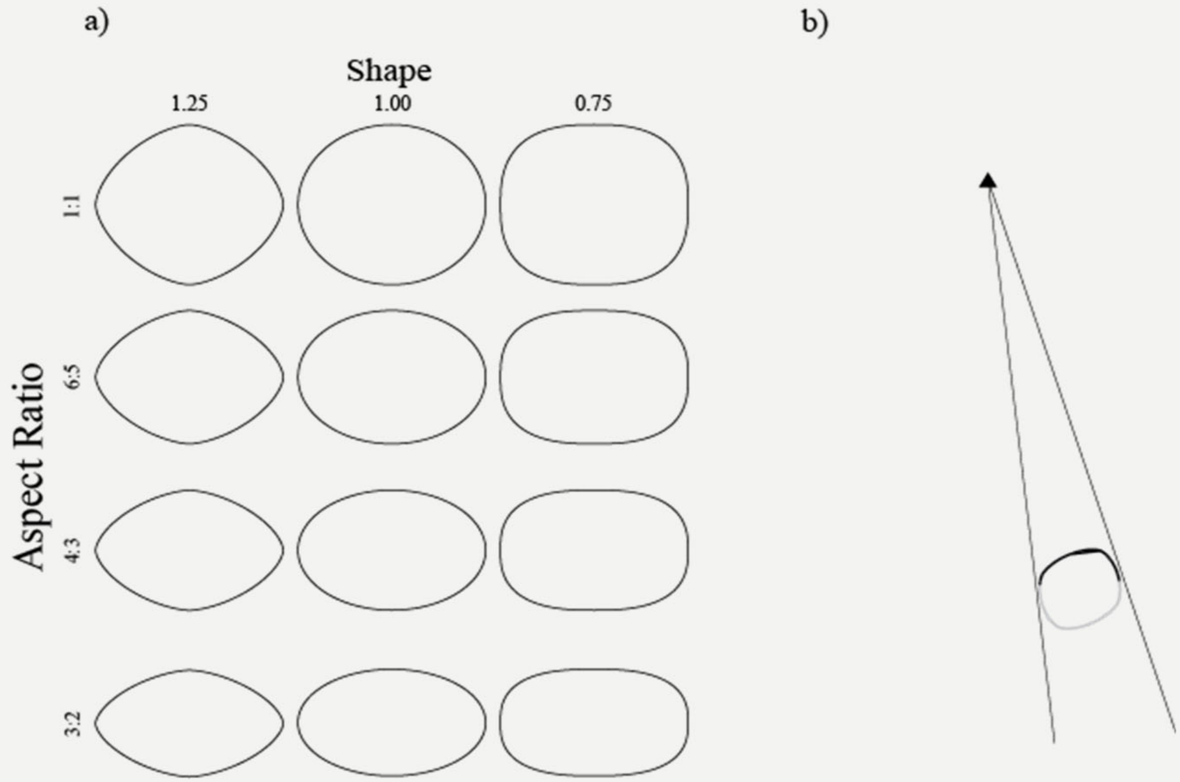
OPTIMIZATION FUNCTION:

$$Q'\left(x, y\right) = \left(1 - w_1 - w_2\right)Q_0^2\left(x, y\right) + w_1 Q_1^2\left(x, y\right) + w_2 Q_2^2\left(x, y\right)$$

w1 and w2 are "arbitrary" weights: w1+w2 < 1

Levenberg-Marquardt minimization algorithm (packaged it all in R)

REGULAR SHAPES WITH SIMULATED OCCLUSION



3 shapes
4 aspect ratios
3 sizes
100 replicates

THE ALGORITHMS

NLS based Circle fitter

Linear transformed Ellipse fitter with Parameter Recovery

Superellipse fitter

ANALYSES

- Examine effects of weights on superellipse fits
  - Identify factors influencing goodness-of-fits
    - Aspect Ratio; Shape; Size; Distance; Orientation; Implied Curvature; ||Arc|| : Circumference; Sector Angle : 360; Sector Area : Shape Area
    - General boosted machine learning to obtain importance scores

- Compare circle fitting, ellipse fitting, and superellipse fitting algorithms
- Compare C, E, and SE algorithms
  - Distributions of errors
  - Identify factors influencing goodness-of-fits

- Error in Cross-Sectional Area estimates

RESULTS: INFLUENCE OF WEIGHTS (SUPERELLIPSE)

- w1 = .1 to .7, .1 steps
- w2 = .1 to .7, .1 steps
- w1 + w2 < 1
- Criteria:
  - # Perfect fits
  - # only perfect fits
  - # first ranks
  - # worse fits
  - # fits with |Error|>100%

- 1767 shape × location × size× rep had perfect fits for all weights
- {.5,.3,.2} had least # of |errors| > 100%
- {.3,.2,.5} had least # of worse fits
- {.3,.1,.6} had greatest # of first ranks
- {.2,.4,.4} had greatest # of perfect fits
- {.2,.2,.6} had the minimum mean rank and greatest number of only perfect fits
- In general, weighting curvature more heavily resulted in better performance

FACTORS INFLUENCING C-S AREA ERRORS BY WEIGHTS

- w0 > .5 (Algebraic Differences)
  - Orientation
  - Implied Curvature

- w1+w2 > .5; w1 ≥ w2 (Gradient)
  - Sector Area : Shape Area

- w1+w2 > .5; w2 > w1 (Curvature)
  - Implied Curvature
  - Sector Angle : 360

NLS CIRCLE FITTING: ERROR DISTRIBUTIONS

Aspect Ratio	Diameter	Shape	Percent Error in Area Estimates						
			< -25	-25 _ < -15	-15 _ < -5	-5 _ < 5	5 _ < 15	15 _ < 25	> 25
3:2	12	1.25	67	4	3	1	0	0	25
		1.00	60	4	3	0	4	6	23
		0.75	59	5	2	0	2	4	28
	24	1.25	61	5	6	1	2	0	25
		1.00	55	4	7	3	1	0	30
		0.75	63	3	3	1	4	2	24
	36	1.25	61	5	7	2	2	1	22
		1.00	61	4	3	4	5	0	23
		0.75	54	7	1	2	2	7	27

ELLIPSE FITTING ALGORITHM: ERROR DISTRIBUTIONS

Aspect Ratio	Diameter	Shape	Percent Error in Area Estimates						
			< -25	-25 – < -15	-15 – < -5	-5 – < 5	5 – < 15	15 – < 25	> 25
3:2	12	1.25	12	6	4	1	5	1	71
		1.00	0	1	1	97	1	0	0
		0.75	62	11	9	0	2	5	11
	24	1.25	11	10	7	1	5	1	65
		1.00	0	1	3	96	0	0	0
		0.75	69	7	1	2	1	0	20
	36	1.25	11	13	1	2	5	2	66
		1.00	1	0	0	97	0	0	2
		0.75	59	6	4	5	1	2	23

SUPERELLIPSE ALGORITHM {.2,.4,.4}: ERROR DISTRIBUTIONS

Aspect	Diameter	Shape	Percent Error in Area Estimates						
Ratio			< -25	-25 _ < -15	-15 _ < -5	-5 _ < 5	5 _ < 15	15 _ < 25	> 25
3:2	12	1.25	6	4	0	86	0	0	4
		1.00	3	1	0	82	0	0	14
		0.75	39	0	0	39	0	0	22
	24	1.25	10	0	2	79	0	0	9
		1.00	3	0	0	85	0	0	12
		0.75	38	0	0	38	0	0	24
	36	1.25	16	1	0	72	1	1	9
		1.00	6	0	0	88	0	0	6
		0.75	45	0	0	32	0	0	23

FACTORS INFLUENCING C-S AREA ERRORS BY ALGORITHM

- NLS Circle
  - Aspect Ratio
  - Implied Curvature
  - Orientation
- Ellipse Fitter
  - Shape
  - Orientation
- Superellipse Fitter
  - Implied Curvature
  - Sector Angle : 360

SUMMARY OF ALGORITHM COMPARISONS

- Circle fits almost perfect when aspect ratio = 1:1
  - Increasingly poor fits when aspect ratio changes
  - Extreme errors (under/over estimation) based on orientation
- Ellipse fits almost perfectly when shape = 1 (smooth circle/ellipse parameter)
- Superellipse consistently better fits across all combinations
  - Flatten ellipses (shape < 1) harder to fit

NEXT STEPS

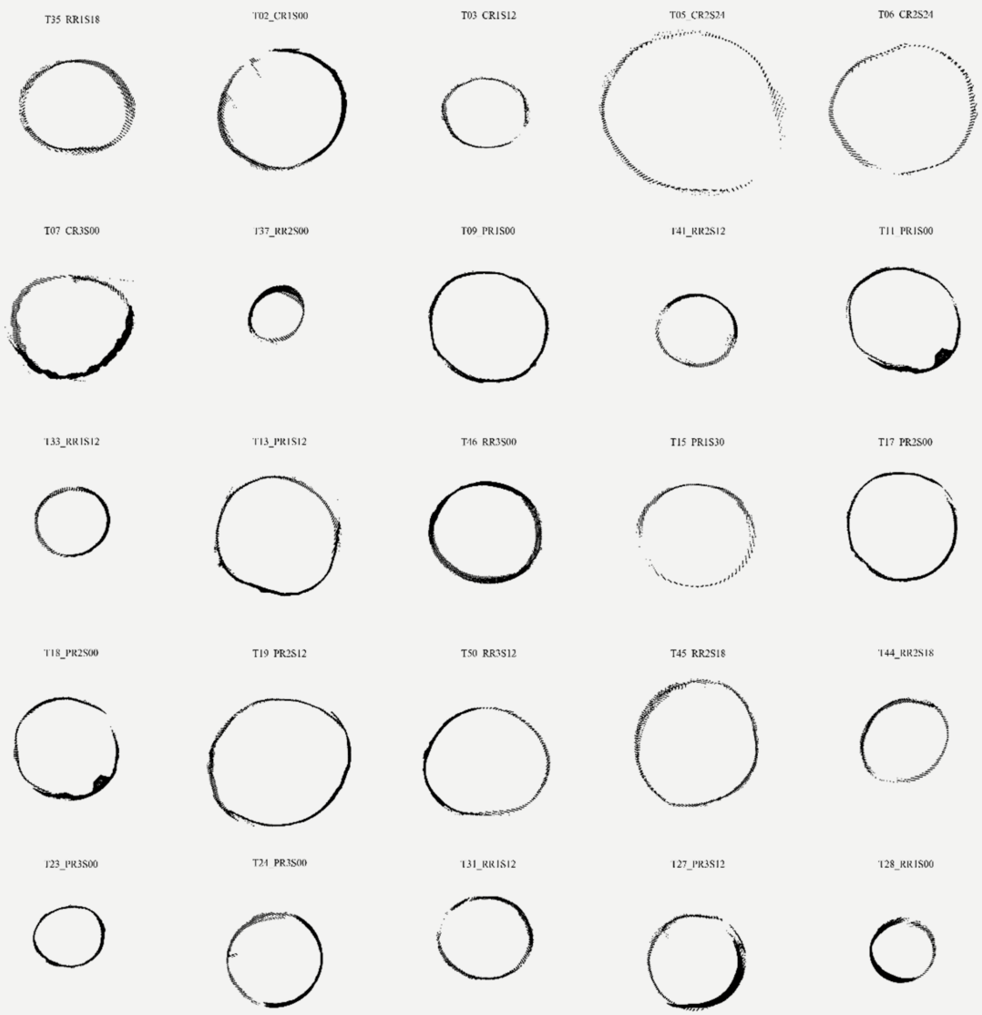
Finish the TLS cross-section analyses

Assess performance under different levels of occlusion

Develop a dynamic weighting algorithm based on implied curvature and other measures of visible shape

Develop a Superellipse-Cylinder Algorithm to fit stem shape

REAL TLS CROSS-SECTIONS



QUESTIONS?